# Poster Abstract: IMPaCT: A Parallelized Software Tool for IMDP Construction and Controller Synthesis with Convergence Guarantees

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## ABSTRACT

In this work, we develop an *open-source* software tool, called IMPaCT, for the parallelized verification and controller synthesis of large-scale stochastic systems using interval Markov chains (IMCs) and interval Markov decision processes (IMDPs), respectively. The tool serves to (i) construct IMCs/IMDPs as finite abstractions of underlying original systems, and (ii) leverage *interval iteration* algorithms for formal verification and controller synthesis over *infinite-horizon* properties, including safety, reachability, and reach-avoid, while offering *convergence guarantees*. IMPaCT is developed in C++ and designed using AdaptiveCpp, an independent open-source implementation of SYCL, for adaptive parallelism over CPUs and GPUs of all hardware vendors, including Intel and NVIDIA. IMPaCT stands as the first software tool for the parallel construction of IMCs/IMDPs, empowered with the capability to leverage high-performance computing platforms and cloud computing services, while providing formal convergence guarantees. We benchmark IMPaCT on several physical case studies adopted from the ARCH tool competition for stochastic models.

## KEYWORDS

Interval Markov decision process, large-scale stochastic systems, automated controller synthesis, parallel construction

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## 1 INTRODUCTION

Large-scale stochastic systems are a crucial modeling framework for characterizing a wide variety of real-world safety-critical systems. Due to the uncountable nature of states and actions in continuous spaces, automating the formal verification and controller synthesis for these complex systems to fulfill some high-level properties, such as *linear temporal logic* (LTL) specifications, is exceedingly formidable. To alleviate this difficulty, one promising solution is to approximate the original system with a simpler model featuring finite state sets, known as *finite abstractions*.

*Interval Markov chains* (IMCs) and *interval Markov decision processes* (IMDPs) [3] have emerged in the literature as a potential finite abstraction for verification and synthesis of stochastic systems fulfilling *infinite-horizon specifications*. In particular, IMDPs incorporate both *lower and upper* bounds on the transition probabilities among finite abstraction partitions. This is accomplished by solving a (multiplayer) game which allows the extension of the satisfaction guarantees to infinite time horizons. However, IMDP constructions are more complicated compared with traditional finite MDPs as it necessitates the computation of both lower and upper probabilistic bounds.

Abstraction-based techniques encounter a significant challenge known as the *curse of dimensionality*, referring to the *exponential growth* in computational complexity as the number of state dimensions increases. To mitigate this, we offer *scalable parallel* algorithms to efficiently construct IMCs/IMDPs and automating the process of verification and controller synthesis over *infinite time horizons*. In particular, by dividing the computations into smaller concurrent operations, we mitigate the overall complexity by a factor equivalent to the number of parallel threads available. This approach enhances the computational efficiency of IMC/IMDP construction, but also facilitates the practical application of these techniques in real-world scenarios with high-dimensional spaces.

**Original contributions.** The primary contributions and noteworthy aspects of our tool include:

(i) We propose the first tool that constructs IMCs/IMDPs for large-scale stochastic systems while providing *convergence guarantees*. Our tool leverages the constructed IMCs/IMDPs for formal verification and controller synthesis ensuring the fulfillment of desired *infinite-horizon* temporal logic specifications, encompassing *safety, reachability, and reach-while-avoid properties*.

(ii) IMPaCT is implemented in modern ISO C++ and runs in parallel using AdaptiveCpp [1] based on SYCL. AdaptiveCpp provides a sturdy foundation for flexible cross-platform parallel processing on CPUs and GPUs from the major hardware vendors such as Intel, NVIDIA, etc.

(iii) IMPaCT leverages the *interval iteration* algorithm to provide *convergence guarantees* to an optimal controller in scenarios with *infinite time horizons*. The trade-off for achieving such a guarantee is the *doubled* computational load compared to the value iteration algorithm.

(iv) IMPaCT accepts bounded disturbances and natively supports additive noises with different *practical distributions* including normal and *user-defined* distributions.


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(v) We leverage IMPaCT across diverse real-world safety-critical applications within *infinite time horizons* such as autonomous vehicles, room temperature systems, and building automation systems. The parallelization outcomes demonstrate significant efficiency in computational time.

The source code for IMPaCT along with comprehensive instructions on how to build and operate it can be located at:

https://github.com/Kiguli/IMPaCT

Details of proposed parallel algorithms, case studies, etc. can be found in the extended version [4].

## 2 INTERVAL MARKOV DECISION PROCESSES

A *continuous-space* MDP can be *finitely abstracted* by an IMDP. Specifically, IMDPs provide bounds over the transition probability of the stochastic kernel $T$, offering a reliable model for analyzing *infinite-horizon* specifications. Although IMDPs incur higher computational costs compared to traditional MDPs, leveraging both lower and upper bound probabilities for state transitions facilitates the resolution of infinite-horizon control problems. It is worth mentioning that since the sets of states $\hat{X}$, inputs $\hat{U}$, and disturbances $\hat{W}$ are all finite, the lower and upper bound transition matrices $\hat{T}_{\min}$ and $\hat{T}_{\max}$ can be represented by static matrices of size $|\hat{X} \times \hat{U} \times \hat{W}|$ by $|\hat{X}|$. This enables the use of powerful iterative algorithms.

**Temporal Logic Specifications.** IMPaCT inherently handles specifications $\psi$ including *safety, reachability, and reach-avoid*, as formally articulated via logical operators *always* $\square$, *eventually* $\lozenge$, and *until* $\mathsf{U}$, in the extended version [4]. When constructing IMCs/IMDPs, we aim at relabeling the states within the state space based on the specification, *e.g.*, safe states, reach states, and avoid states. This is especially beneficial for later-stage verification or controller synthesis processes.

## 3 PARALLEL CONSTRUCTION OF IMDP

IMPaCT supports the parallel construction of IMDPs. This IMDP construction is required for our controller synthesis approach discussed in Section 4. The details of the proposed parallel algorithm can be found in the extended version [4, Algorithm 2].

**Complexity Analysis.** Parallelization offered by IMPaCT enhances the performance of solving both abstraction and synthesis problems. Using $O(\kappa)$ to represent the nonlinear optimization complexity required for the IMDP construction, the computational complexity of our parallel algorithms is $O(\frac{2\kappa d}{\text{THREADS}})$, where $d = (n_{x_i}^n \times n_{u_j}^m \times n_{w_k}^p) \times n_{x_i}^n$, with $n_{x_i}$, $n_{u_j}$, and $n_{w_k}$ being the dimension-wise counts of partitions within $\hat{X}$, $\hat{U}$, and $\hat{W}$, respectively, where $i = 1, \ldots, n$, $j = 1, \ldots, m$, and $k = 1, \ldots, p$, and THREADS is the number of parallel threads running. Given that the construction involves both $\hat{T}_{\min}$ and $\hat{T}_{\max}$, the computational complexity $d$ is doubled.

## 4 PARALLEL CONTROLLER SYNTHESIS WITH INTERVAL ITERATION

IMPaCT synthesizes a controller, via the constructed IMDP, to enforce both *finite* or *infinite-horizon* properties over the dt-SCS.

**Interval Iteration.** When dealing with infinite-horizon problems, the *interval iteration* algorithm is capable of providing convergence guarantees unlike the common *value iteration* [2]. In particular, the interval iteration algorithm converges to an under-approximation and over-approximation of the satisfaction probability associated with a temporal logic specification $\psi$. The trade-off for achieving such a guarantee is the doubled computational load compared to the value iteration algorithm. In this case, the algorithm parameters for IMDPs, known as the *feasible distribution*, must first be calculated via dynamic programming.

| Case Study | Spec | $\lvert\hat{X}\rvert$ | $\lvert\hat{X} \times \hat{U} \times \hat{W}\rvert$ | CPU i9 | GPU RTX |
|---|---|---|---|---|---|
| 2D Robot | R | 441 | 53,361 | 2.26 | 5.2 |
| 2D Robot | R − A | 1,681 | 741,321 | 577 | 216 |

**Table 1: Execution times in seconds for controller synthesis, comparing both a CPU (Intel i9-12900) and a GPU (NVIDIA RTX A4000) for a 2D Robot including specifications of both reachability (R) and reach-avoid (R − A) properties.**

**Parallel Controller Synthesis via IMDPs.** We propose parallel algorithms [4, Algorithms 5 & 6] to enhance the computational efficiency of controller synthesis procedure acquiring the resulting controller $C := (\mathcal{S}, \pi, \mathbb{P}_{\psi_{\min}}, \mathbb{P}_{\psi_{\max}})$, consisting of initial states $\mathcal{S}$, optimal inputs $\pi$, lower bound satisfaction probabilities $\mathbb{P}_{\psi_{\min}}$, and upper bound satisfaction probabilities $\mathbb{P}_{\psi_{\max}}$. In particular, the synthesis process includes solving a *multiplayer game* for the input selection. We also employ the *sorting approach* from [3, Lemma 7] for more efficient synthesis.

**Parallel Verification Analysis via IMCs.** In striving for maximum user-friendliness, IMPaCT *automatically* distinguishes between verification using an IMC and synthesis via an IMDP. In particular, for verification the users only need to specify the absence of input parameters. The corresponding generated lookup table is then $C := (\mathcal{S}, \mathbb{P}_{\psi_{\min}}, \mathbb{P}_{\psi_{\max}})$.

## 5 BENCHMARKS AND CASE STUDIES

We illustrate IMPaCT's applications by employing multiple well-known benchmark systems, encompassing safety, reachability, and reach-avoid specifications over *infinite time horizons*. Our benchmarks encompass several complex physical case studies, including a 2D robot, a 3D autonomous vehicle, 3D and 5D room temperature control systems, as well as 4D and 7D building automation systems. We also consider a 14D case study used for the purposes of showing the scalability of the tool. In Table 1, we compare synthesis times for CPU against GPU, demonstrating larger benchmarks have notable efficiency improvements with GPU synthesis. In the extended paper [4, Table 1], we showcase the underlying details of these case studies, including memory usage and computation times.

## REFERENCES

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